**Homework 3**



**P3.1.16** Determine each of the resistors of the equivalent delta between terminals ‘a’, ‘b’, and ‘c in Figure P3.1.16, expressed in S.

**Solution:** The inner Δ is transformed to three 2/3 Ω resistors connected in Y. To each is added 1/3 Ω in series to give three 1 Ω resistors connected in Y. These are transformed to three 1/3 S resistors connected in Δ. Each of the resistors of the equivalent Δ will be (1/3 + 1/6) = 1/2 S.



**P3.1.18** Determine *Rin* in Figure P3.1.18.

**Solution:** Apply a test voltage source *VT*; *Rin* = *VT*/*IT*. The current in *R*/2 is 2*VT*/*R*. From KVL around the mesh in the middle, starting at node ‘b’ and going CCW: +2VT – *Vca* – *VT* = 0, which gives *Vca* = *VT*. The current through *R* in the middle branch is *VT*/*R* directed upwards. From KVL around the outer loop, starting at node ‘b’ and going CCW: -2*VX* + *VX* – *VT* = 0, which gives *VX* = -*VT*. The current in *R* on the RHS is also *VT*/*R* directed upwards. From KCL at node ‘a’: *IT*+ *VT*/*R* + *VT*/*R* – 2*VT*/*R*. This makes *IT* = 0, so that *Rin* = *VT*/*IT* → ∞.



**P3.2.11** Determine *IO* by successive current division at nodes ‘c’, ‘b’, and then ‘a’ in Figure P3.2.11.



**Solution:** The resistance between nodes ‘c’ and ‘d’ is 0.6||3 = 0.5 kΩ;



From Ohm’s law 0.5×*Ibc* = 0.6×*IO* so that *Ibc* = 1.2*IO*. The resistance to the right of node ‘b’ is (1.5 + 0.5) = 2 kΩ. Hence, *Iab* = 2*Ibc* = 2.4*IO*. The resistance to the right of node ‘a’ is (4 + 2||2) = 5 kΩ, which is one-half of 10 kΩ. Hence, *Iab* = (2/3)×30 mA = 20 mA. equating this to 2.4IO, gives *IO* = 20/2.4 = 200/24 = 25/3 mA.



**P3.2.15** Determine *IO* in Figure P3.2.15 using source transformation.

**Solution:** The 10 V source in series with 2 Ω is transformed to a 5 A source in parallel with 2 Ω. The 3*IX* source in series with 5 Ω is transformed to a 0.6*IX* current source in parallel with 5 Ω. The 5 Ω is combined in parallel with 2 Ω to



give a parallel resistance of 10/7 Ω. The 0.6*IX* current source in parallel 10/7 Ω is transformed to a voltage source of 6*IX*/7 in series with 10/7 Ω. The 10/7 Ω in series with 2 Ω is a resistance of 24/7 Ω. The 6*IX*/7 voltage source in series with 24/7 Ω is transformed to a current source of 0.25*IX* in parallel with 24/7 Ω. The circuit becomes as shown.

From KCL: 5 + 3*IX*/2 + *IX* + 7*IX*/8 = 0.25*IX*, or 40 + 12*IX*  *+* 8*IX* + 7*IX* = 2 *IX*. *This* gives *IX* = -40/25 = -1.6 A; *IO* = 7*IX*/8 – 0.25*IX* = 0.625*IX* = -1 A.

**P3.2.16** Determine *VY* in Figure P3.2.16.



**Solution: Initialize.** All given values and the required *VY* are entered.

**Deduce.** **Step 1:** From KCL the current between the 4 Ω and 8 Ω resistors is zero. **Step 2:** This means that *VY* can be obtained by voltage division as:  V.



**P3.3.1** Determine *ISRC* in Figure P3.3.1 so that no current flows in 

**Solution: Initialize.** The given values are entered, the nodes are labelled, and zero current is assumed to flow.

**Deduce.** **Step 1:** Because of the zero current, the same current passes through the 3 kΩ and 1 kΩ resistors. Hence, voltage division applies, and  V. **Step 2:** *Vbc* = 0, since no current flows in *RL*. **Step 3:** From KCL at node ‘c’, *ISRC* flows through the 3 kΩ resistor, so that *Vcd* = 3*ISRC*. **Step4:** Applying KVL around the mesh ‘dabc’ starting at node ‘d’ and going CW: 3 + 6 + 0 – 3*ISRC* = 0, which gives *ISRC* = 3 mA.



**P3.3.7** Determine *VX* in Figure P3.3.7.



**Solution:** **Initialize.** All given values and the required *VX* are entered. The nodes are labelled.

**Simplify.** No meaningful simplifications can be made.

**Deduce.** **Step 1:** If the upper mesh is enclosed by a, 9 A enter this figure from the left, and 3 A leave the figure through the source. It follows from KCL that 6 A should flow through the 3 Ω resistor. **Step 2:** voltage drop across the 3 Ω resistor is 18 V in the direction of the 6A.



**Explore.** To derive *VX*, the voltage across the 4 Ω resistor is needed. If an unknown current *I* is assigned through this resistor, the current in the upper branch is, from KCL at node ‘a’: (9 – *I*); *I* can then be found from KVL around the mesh. Starting at node ‘a’ and going CW; -30 -2(9 – *I*) + 4*I* = 0. This gives *I* = 8 A. It follows that *VX* = 4×8 + 18 = 50 V.

**Check.** The 30 V source in series with 2 Ω is transformed to a 15 A source in parallel with 2 Ω; the parallel combination of this resistor and the 4 Ω resistor is a resistor of Ω. From KCL at node ‘a’, the current in the 4/3 Ω resistor is 24 A, and the *Vab* = 32 V. Adding this to 18 V gives *VX* = 50 V, as before.



**P3.3.8** Determine *VO* in Figure P3.3.8.



**Solution:** **Initialize.** All given values and the required *VO* are entered. The nodes are labelled.

**Deduce.** **Step 1:** *Vbc* = 4*IO*. **Step 2:** Current in 4 Ω resistor is (4*IO* – 12)/4 = (*IO* – 3) A. **Step 3:** Current entering node ‘b’ from the 2 Ω resistor in the middle is (2*IO* – 3) A. **Step 4:** Current entering node ‘a’ from the 2 Ω resistor on the left is (2*IO* – 3 – 3*IO*) = -(*IO* + 3) A. **Step 5:** *IO* can be determined from KVL around the outer loop. Starting at node ‘c’ and going CW: +24 + 2(*IO* +3) – 2(2*IO* – 3) – 4*IO* = 0, or 36 – 6*IO* = 0, which gives *IO* = 6 A, and *VO* = 12 V.



**Check.** The circuit can be reduced to a two-essential-node circuit by source transformation. The 24 V source in series with 2 Ω is transformed to a 6 A source in parallel with 2 Ω. The 12 A source is combined with the 3*IO* source to give a source of (3*IO +* 12) = 3(*IO +* 4) A in parallel with 2 Ω. The current source is transformed to a voltage source 6(*IO +* 4) V and the two 2 Ω sources combined into a 4 Ω resistor. The circuit becomes a shown. KCL at node ‘b’: , or 3*IO* + 12 – 2*IO* = 4*IO* – 6, or 3*IO* = 18, and *IO* = 6 A, as before.



**P3.3.11** Determine *VY* in Figure P3.3.11.



**Solution: Initialize.** All given values and the required *VY* are entered. The nodes are labelled. It is seen that this is a two-essential-node circuit.

**Simplify.** The 30 Ω and 20 Ω resistors are combined into a 50 Ω resistor. The 30 Ω and 15 Ω resistors are combined into a 45 Ω resistor. The circuit is redrawn to show it more clearly as a two-essential-node circuit.



**Deduce.** *Vab* = 50*IX*; The current in the 30 V and 45 Ω branch is:   A. From KCL at node ‘a’: 10 – *IX* + 2 = , or 108 – 9*IX* = 10*IX –* 6, or 19*IX* = 114, which gives: *IX* = 6 A. It follows that *Vab* = 50×6 = 300 V = *VY* – 40×2 + 40×6, or *VY* = 300 – 160 = 140 V.